Normal Transmission-Line Networks and Their Lumped LC Presentation

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Abstract—**A previous result [5] is extended and a new presentation is described for Richards' transmission-line (TL) networks. The new presentation is in a lumped *LC* form; therefore, classical analysis and synthesis techniques are directly applicable.**

The class of networks for which this presentation is possible is studied in detail. This class of networks, called normal TL networks, is found to be a necessary and sufficient condition for driving-point immittance functions to be rational positive real functions of $\lambda = \tanh \tau p$.

The effectiveness of this representation is further demonstrated by applying it to graph-transformation analysis method. It simplifies the procedure considerably and reveals additional physical insight into the TL network.

I. Introduction

RICHARDS' resistor-transmission-line (TL) network [1] is essentially a hybrid distributed and lumped network. The structure is distributed, but in the analysis, the TL junctions are idealized into nodes. In frequency plane $\lambda = \tanh \tau p$, the TL stub may be regarded as a lumped inductor or capacitor; but as a two-port, a unit length of TL (unit line) is not a lumped component. The irrational factor $\sqrt{1-\lambda^2}$ in its transfer function renders it impossible to be represented by finite lumped network.

If the unit length of TL is subdivided and $\lambda' = \tanh \tau p/2$ regarded as the fundamental frequency, a unit line can be presented by a lattice *LC* section (alternatively, a Darlington *C* section) [1]. It is therefore possible to present Richards' TL networks completely by lumped components. However, such a presentation complicates the network structure and doubles the number of elements in the presentation. This is not acceptable to the circuit designer. To date, most TL networks have been designed using λ as the fundamental frequency variable.[1]

Thus earlier workers were resigned to the idea and treated the unit line as a new "basic" component. Numerous new analytic and synthesis techniques were discovered which could accommodate this new element. Accompanying these, a parallel set of TL network theories has also been developed in order to justify the validity of each synthesis process.

Closer examination of this treatment reveals a disturbing feature. On the one hand, lossless TL networks consist of *three* basic components: *L*, *C*, and the unit line. On the other hand, the driving point (DP) immittance functions of most TL networks are positive real (PR) *rational* in λ, in no way different from those of a *two*-component lumped *LC* network. The new basic element, the sole existence of which distinguishes a TL network from a lumped network, does not seem to leave any imprint in the DP immittance functions. This leads one to suspect that perhaps the new basic component is not necessary; perhaps most lossless TL networks can be adequately presented by *two* basic components only: *L* and *C*.

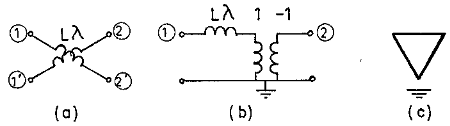

Fig. 1. (a) A balanced TL network. (b) Its unbalanced equivalent network. (c) A triangular TL loop.

This, in fact, is the case. Under a simple congruent transformation, it will be shown in this paper that lossless TL networks may be presented by a lumped *LC* network in the frequency domain $s = \sinh \tau p$.[2] This immediately dissolves the seemingly contradictory feature discussed previously.

The implication of this *s*-plane presentation does not end here. As the resulting network has a completely lumped presentation, it inherits the entire wealth of classical network theory. This includes numerous lumped synthesis procedures developed in the past four decades. Moreover, if we were to carry each TL network synthesis technique in turn using *s*-plane presentation, more often than not we could identify it with an existing lumped network synthesis technique. This is discussed to some extent in [5]; it is thus omitted here.

In this paper, it will be demonstrated that *s*-plane presentation is equally efficient in graph-transformation techniques [3], [4]. It reduces the number of necessary steps and sometimes carries the simplifying process further than previously deemed possible.

Before this is done, however, the class of networks for which the new presentation is possible will be studied. This class of networks is a generalized version of Ikeno's *normal* TL networks [6], [10]. It will now be defined in precise terms.

II. Normal Resistor TL Networks

A. Definition

The class of TL networks under consideration here consists of resistors, unit lines, short-circuited and open-circuited stubs, and multiwire lines. The lines are uniform, lossless, and of equal length. We assume that TL networks, either in balanced form or unbalanced form, have an equivalent ground plane (or equipotential surface). All accessible ports must have one terminal on the common ground plane. The resulting *n*-port network thus behaves like an $(n+1)$ terminal network. The method of admittance matrix addition for parallel connection is always valid.[3] We shall disregard the ground plane and represent the unit line graphically by a thick line. The two-port structures such as the one in Fig. 1(a) will be

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[1] A notable exception is in the design of a branch-guide coupler [2]. It uses $\frac{1}{8}$ of a wavelength instead of the customary $\frac{1}{4}$ wavelength in designing the symmetrical half-section. However, the overall structure still has the prime unit of $\frac{1}{4}$ wavelength.

[2] This transformation was first used by Seidal and Rosen [20]. The application, however, was very different from that intended to be given here.

[3] See Weinberg [7, p. 20] for validity test.

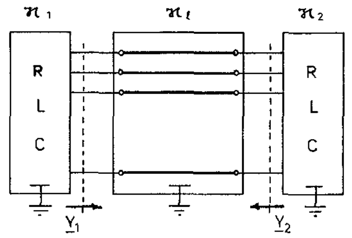


Fig. 2. The n-port presentation of normal TL networks.

excluded from consideration here because, by itself, the network requires an extra ideal transformer in its unbalanced presentation [Fig. 1(b)]. This would complicate the discussion in later sections.

In the case of a multiwire-line structure, it may be more convenient to decompose it into an interconnection of short-circuited stubs and unit lines with positive and negative impedances. This presentation was recently discovered by Sato and Cristal [3].

Given such a TL network, if we: 1) decompose the multiwire line (if any) into the uncoupled form of Sato and Cristal, 2) remove all shunt two-terminal components that are connected to the common ground plane, and 3) short circuit all other series two-terminal components, we obtain a skeleton network consisting of positive uncoupled unit lines only. Its line presentation is a linear graph [8]. This graph may be hinged, and may contain a self-loop, but it must be connected. We shall henceforth refer to this graph as the TL graph of the network.

We now extend Ikeno's definition and define the following.

Definition: A TL network is *normal* if its TL graph is connected and contains no loop consisting of odd numbers of unit lines.

This structural restriction is reflected in the terminal characteristic of a normal TL network. In the following sections, we shall show that this is, in fact, a necessary and sufficient condition for its DP immittance to be a PR *rational* function in λ.

B. n-Port Presentation of Normal TL Networks

Given a normal TL network, the following procedure will transform it into a form shown in Fig. 2.

1) Number the nodes consecutively along the TL part of the network.

2) Move all the odd nodes to one side and the even nodes to the other side.

The transformation will be successful if the TL network is normal. This can be readily proven by the graph-theoretical approach [9]. In Fig. 2, network $\mathfrak{N}_l$ consists of n unit lines, which may or may not be coupled together. The terminating networks $\mathfrak{N}_1$ and $\mathfrak{N}_2$, on the other hand, consist of λ-plane *RLC* components only. In many cases, they consist simply of zero-length short-circuiting wires joining the nodes together. We can now state the first theorem of interest.

Theorem 1: Given a normal TL network with *all* its accessible ports on one side of $\mathfrak{N}_l$, the immittance matrix defined with respect to these ports is a PR *rational* matrix of λ.

In particular, the DP immittance of a normal TL network at *any* node is a PR rational function of λ. This fact was recognized by many authors [10], and the proof has been given for a less general form of normal TL networks. In its general form, Theorem 1 can be proven readily by n-port scattering matrix formulation [11], [12]. This is given in Appendix I.

C. Time-Domain Scattering Interpretation of Normal TL Networks

Normal TL networks can be distinguished from other TL networks very distinctly by time-domain scattering matrix interpretation. If an incident wave of narrow pulse width ($<\tau$) is launched from a sending node, the response at any receiving node is a sequence of similar pulses of various heights and polarity. The pulses in the sequence correspond to various TL routes taken, and their arrival time is proportional to the route length. If we follow each route and assign a pulse (P_i) for every route (R_i) taken, then every pulse in the response sequence can be accounted for. This concept is very well known and needs no further elaboration.

Since the unit lines in the network are commensurate in length, the pulses will arrive at discrete intervals of time. If the network is normal and the sending node and receiving node are at the same side of $\mathfrak{N}_l$ (Fig. 2), all TL routes joining these two nodes must consist of even numbers of unit lines. It follows that pulses must arrive at time $t=2i\tau$, $i=0, 1, 2 \cdots$. If the sending node and receiving node are at opposite sides of $\mathfrak{N}_l$, then pulses will arrive at odd intervals of time $t=(2i+1)\tau$, $i=0, 1, 2 \cdots$. In both cases, the response pulses are spaced even numbers of τ intervals apart.

If the TL network is not normal, the above phenomenon is no longer true. This is stated in definite terms as follows.

Theorem 2: If a TL network is not normal, the impulse response at any node includes two impulses of finite energy spaced odd numbers of τ intervals apart.

The proof of this theorem is given in Appendix II. Theorems 1 and 2 are now combined to obtain the main theorem of interest.

Theorem 3: The driving-point immittance function of a TL network at any node is a PR *rational* function of λ if and only if the network is *normal.*

Proof: Sufficient proof follows directly from Theorem 1. To prove the necessary condition, we let $z(\lambda)$ be the rational immittance function. Substituting in

$$\lambda = \frac{1 - e^{-2\tau p}}{1 + e^{-2\tau p}}$$

we can express $z(\lambda)$ in power series form

$$z(\lambda) = \sum_{n=0}^{\infty} C_n e^{-2n\tau p} \tag{1}$$

where C_n are real coefficients.

Taking the inverse Laplace transform, we obtain

$$\mathcal{L}^{-1} z(\lambda) = \sum_{n=0}^{\infty} C_n \delta(t - 2n\tau) \tag{2}$$

where $\delta(t)$ is a Dirac impulse function. Equation (2) asserts that the DP impulse responses of the TL network are sequences of impulses spaced 2τ apart. If the TL network were to contain an odd TL loop, Theorem 2 would contradict this assertion. The network, therefore, must be normal.

Q.E.D.

Theorem 3 explains why all practical TL networks designed in the past are normal. The designs were based on lumped insertion-loss synthesis theory; the DP immittance of the resulting network must by necessity be rational in λ. This

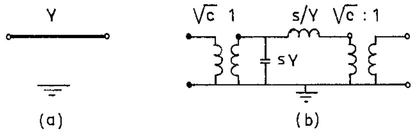


Fig. 3. The decomposition of a unit line.

class of networks includes stepped-impedance transformer, cascade microwave filters, interdigital filter, directional coupler, branch-guide coupler, meander-line network. None of these networks contain the odd TL loop of Fig. 1(c) as a subnetwork.

III. Lumped LC Presentation of Lossless TL Networks

In Section II, the properties of normal TL networks are studied and the motivation behind such a distinction is shown. In this section, the main aim of this paper will be developed as follows.

A. Decomposition of Unit Line

In Fig. 3, the unit line is decomposed into four subcomponents. The two ideal transformers with turns ratio $\sqrt{\cosh \tau p}$ are fictitious elements created for mathematical convenience. The chain matrix of the composite network, given by

$$\boldsymbol{C} = \begin{pmatrix} \sqrt{c} & 0 \\ 0 & 1/\sqrt{c} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ sY & 1 \end{pmatrix} \begin{pmatrix} 1 & sZ \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \sqrt{c} & 0 \\ 0 & 1/\sqrt{c} \end{pmatrix} = \begin{pmatrix} c & Zs \\ Ys & c \end{pmatrix} \tag{3}$$

is identical to that of a unit line, thus proving its equivalency as a two-port. The c and s in (3) are the shorthand notations for $\cosh \tau p$ and $\sinh \tau p$, respectively. Note that in the equivalent presentation, s is used as the fundamental frequency in preference over Richards' frequency variable $\lambda = \tanh \tau p$. Note also that the equivalent network is structurally asymmetrical. We shall label the node at the capacitance end by a black dot and call it a capacitance node, and the node at the inductance end by a small circle and call it an inductance node. Such a distinction is purely artificial; we can also validly present the same unit line by the inverted network of Fig. 3(b).

There are also two alternative ways of presenting TL stubs. Referring to Fig. 3, a short-circuit stub can be presented either as an inductor or a parallel LC in cascade with an ideal transformer, depending on which end is short circuited. Similarly, an open-circuit stub can be presented as a capacitor or series LC in cascade with an ideal transformer.

By using such an equivalent presentation, any lossless TL networks consisting of unit lines, multiwire lines, and stubs can be converted into a lumped LC network imbedded in a bank of ideal transformers of turns ratio $\sqrt{c}$.

B. Separation of Lumped LC Network

The decomposition process appears to complicate the networks, were it not for the fact that ideal transformers may be separated from the circuit. Consider a typical TL junction shown in Fig. 4, in it the unit lines are presented by their equivalents with capacitance ends facing one another. Since the transformers have the same turns ratio, they can be removed and replaced by a single transformer, as shown. If all ideal transformers were removed in this manner, the resulting network would be a lumped LC network. Note also that the shunt capacitances and series inductances can be recombined between adjacent elements, thus greatly simplifying the resulting network. We shall henceforth denote the lumped LC network by $\mathfrak{N}$ and call it the s-plane equivalent of the original TL network $\bar{\mathfrak{N}}$.

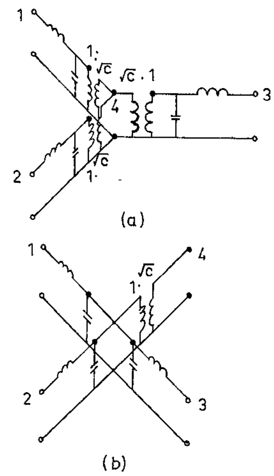


Fig. 4. Separation of ideal transformer from a TL junction.

Not all lossless TL networks can be converted to a lumped LC network in the manner described. Any attempt to remove all the ideal transformers from the equivalent network of the triangular loop in Fig. 1(c) will end in frustration. The following theorem states the sufficiency condition for such a presentation to be possible.

Theorem 4: All normal lossless TL networks $\bar{\mathfrak{N}}$ have a lumped s-plane presentation $\mathfrak{N}$.

Proof: The proof is simple. Given a lossless normal TL network with an n-port presentation (Fig. 2, without the resistors), we can label the nodes in $\mathfrak{N}_1$, as inductance nodes, and the nodes in $\mathfrak{N}_2$ as capacitance nodes. When the equivalent networks are substituted according to the node notation, all ideal transformers will face the same way and can be removed. The remaining network is the lumped LC network that we sought. Q.E.D.

If we renumber the inductance nodes first $(1, 2, \cdots, m)$, and capacitance nodes next $(m+1, \cdots, n)$ and let $\boldsymbol{Y}$ and $\bar{\boldsymbol{Y}}$ denote the admittance matrices of $\mathfrak{N}$ and $\bar{\mathfrak{N}}$, respectively, Theorem 4 can be stated algebraically as follows.

Theorem 4(a): The congruent transformation

$$\boldsymbol{Y} = \boldsymbol{N}'\bar{\boldsymbol{Y}}\boldsymbol{N} \tag{4}$$

where $\boldsymbol{N} = \text{diag}\,[1/\sqrt{c}, 1/\sqrt{c}, \cdots, \sqrt{c}, \sqrt{c}, \cdots]$ transforms the matrix $\bar{Y}(\lambda)$ into a lossless positive real rational matrix $\boldsymbol{Y}(s)$ in s.

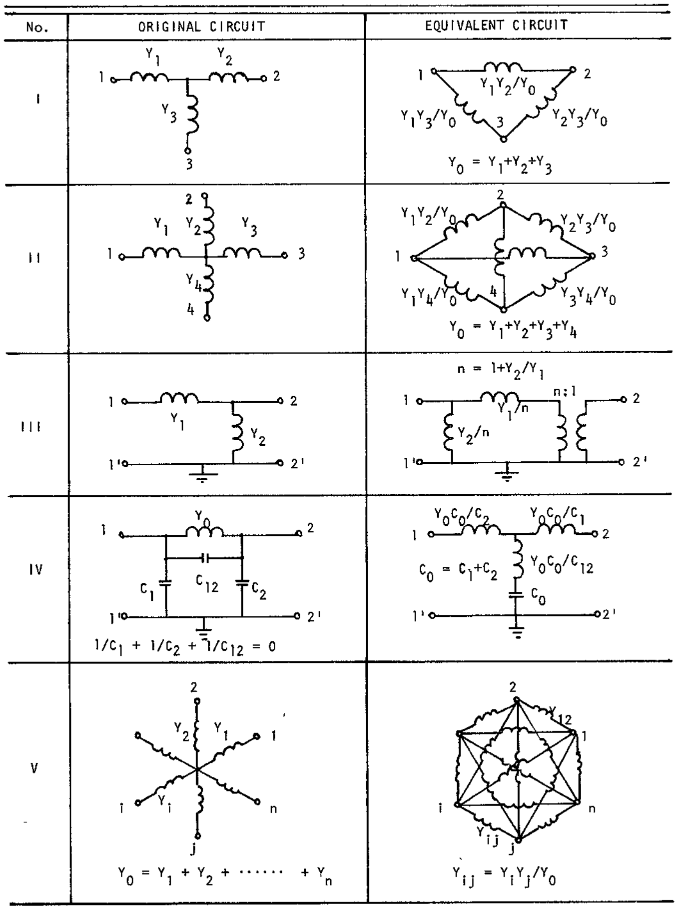
Unlike ordinary congruent transformation ($\boldsymbol{N}$ real), (4) alters the network structurally rather than impedance-level wise.

C. $\sqrt{c}$ Transformers

The bank of $\sqrt{c}$ transformers in which $\mathfrak{N}$ is imbedded may appear to be difficult to handle. However, it does not prevent us from analyzing or synthesizing a normal TL network $\bar{\mathfrak{N}}$ via its s-plane equivalent $\mathfrak{N}$.

In most applications, all terminals except input and output ports are floating terminals. The open-circuited ideal transformers at these floating terminals can be disregarded.

TABLE I
NETWORK IDENTITIES



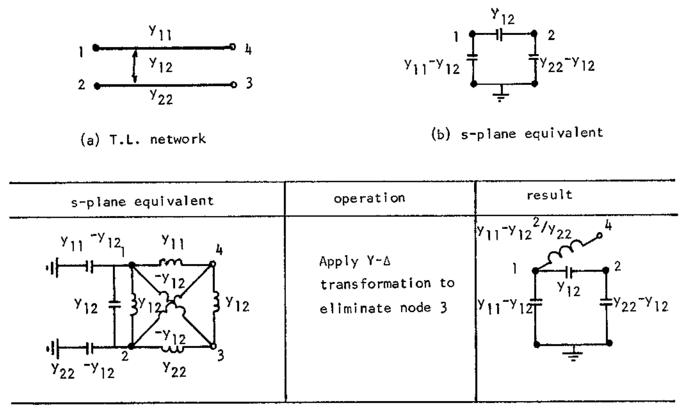


Fig. 5. Equivalent circuit derivation.

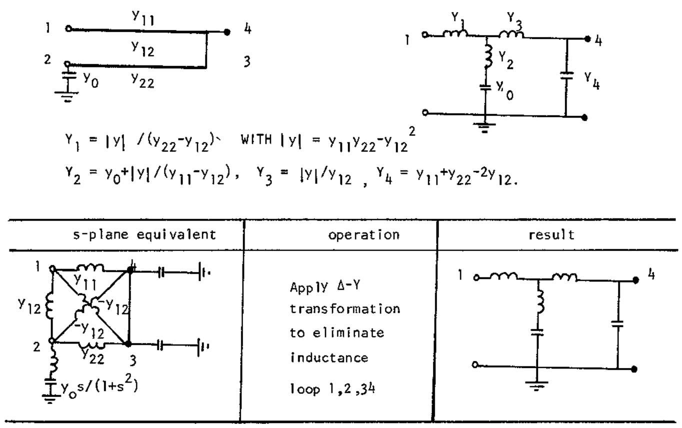


Fig. 6. Equivalent circuit derivation.

While the remaining $\sqrt{c}$ transformers at the input and output ports cannot be disregarded in this manner, they can be circumvented as follows.

Consider the two-port case (single input and single output). Equation (4) becomes simply[4]

$$\begin{pmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{pmatrix} = \begin{pmatrix} \bar{y}_{11}/c & \bar{y}_{12} \\ \bar{y}_{21} & c \cdot \bar{y}_{22} \end{pmatrix}. \quad (5)$$

Following the classical insertion-loss synthesis procedure, we first derive the scattering parameters of $\overline{\mathfrak{N}}$ algebraically from a specification of transducer power gain, then obtain the short-circuited driving-point admittance from these scattering parameters [16, eq. (79)]

$$\bar{y}_{11}(\lambda) = \frac{\bar{g}_e(\lambda) - \bar{h}_e(\lambda)}{\bar{g}_0(\lambda) + \bar{h}_0(\lambda)} \text{ or } \frac{\bar{g}_0(\lambda) - \bar{h}_0(\lambda)}{\bar{g}_e(\lambda) + \bar{h}_0(\lambda)} \quad (6)$$

where

$\bar{s}_{11}(\lambda) = \bar{h}(\lambda)/\bar{g}(\lambda)$
$\bar{g}_e(\lambda)$ = even part of polynomials $\bar{g}(\lambda)$
$\bar{g}_0(\lambda)$ = odd part of polynomials $\bar{g}(\lambda)$
etc.

are Belevitch's notations for scattering parameters of lossless two-ports terminated in 1-Ω resistors [16].[5]

[4] Assume that port 1 is labeled as a capacitance node and port 2 as an inductance node.
[5] Note that $\bar{y}_{11}(\lambda)$ and $\bar{s}_{11}(\lambda)$ are *rational* functions in λ even if the transfer function $\bar{y}_{21}(\lambda)$ may contain an irrational factor $\sqrt{1-\lambda^2}$.

The s-plane network $\mathfrak{N}$ can now be realized from (5), i.e., from

$$y_{11}(s) = \bar{y}_{11}(\lambda)/c. \quad (7)$$

Finally, $\overline{\mathfrak{N}}$ is recovered from $\mathfrak{N}$ by a reverse decomposition process. In this way, the $\sqrt{c}$ transformers are circumvented in the realization process.

The details of this synthesis technique have been presented in a slightly different form in [5]; therefore, they will be omitted here. In this paper, we are mainly concerned with another aspect of s-plane presentation, viz., the graph-analytical techniques of TL networks via their s-plane equivalents.

IV. Graph-Transformation Technique in s-Plane

Because of their unique structure, TL networks in the past tended to develop graph-transformation techniques of their own. Examples are Kuroda identities, and those by Sato and Cristal [3] and Pang [13].

Since the s-plane network $\mathfrak{N}$ consists of only two basic components as compared to three in λ-plane presentation, one would expect the new presentation to be more amiable to graph-transformation technique. This, in fact, is the case, as will be illustrated by a few examples. Some examples in [3] will be reconsidered here for comparison purposes. In all the examples, the final s-plane equivalents were obtained by the procedure as follows.

1) Label the nodes on the TL network alternatively as capacitance nodes and inductance nodes.

2) Substitute in the lumped LC equivalent (Fig. 3, without the transformers) for each component.

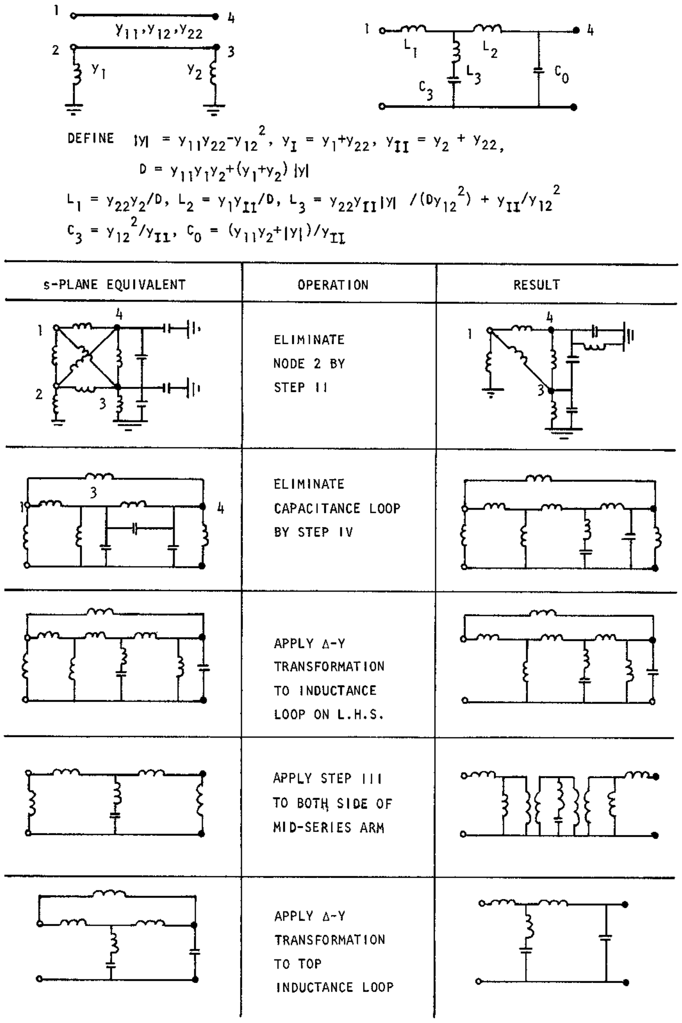


Fig. 7. Equivalent circuit derivation.

3) Apply various transformations listed in Table I.

It is interesting to compare the identities in Table I with their counterparts in [3]. Steps 1 and 2 [3, table I] are similar to the lumped Y–Δ transformation (step I); steps 4 and 5 [3, table I] are similar to the "cross–diamond" transformation (step II); Kuroda identities [3, table II, steps 2 and 6] are similar to the Norton transformation (step III). The T-pi transformation in step IV of this paper shows the parallel and series form of Darlington C section, of which the network in [3, table II, step 1] are special cases. Finally, Sato's two universal transformations [21] are similar to the node elimination of step V, of which steps I and II are special cases. These identities are commonplace in the context of lumped analysis, but nontrivial in the form of TL networks.

Figs. 5 and 6 in this paper consider the same examples as shown in [3, figs. 5 and 7]. Both s-plane equivalents are obtained by a single Y–Δ transformation. Note that we label 1 and 2 as capacitance nodes in Fig. 5, while 1 and 2 are labeled as inductance nodes in Fig. 6. In Fig. 7,[6] the Brune section of Rhodes, Scalan, and Levy is transformed into its s-plane equivalent. This is not a trivial exercise.

Fig. 8 shows a simple double-loop two-port network. Its s-plane equivalent was obtained by applying Y–Δ transforma-

[6] Note the typographical error in the expression for Y_0 [3, fig. 8]. Y_0 and C_0 should be identical.

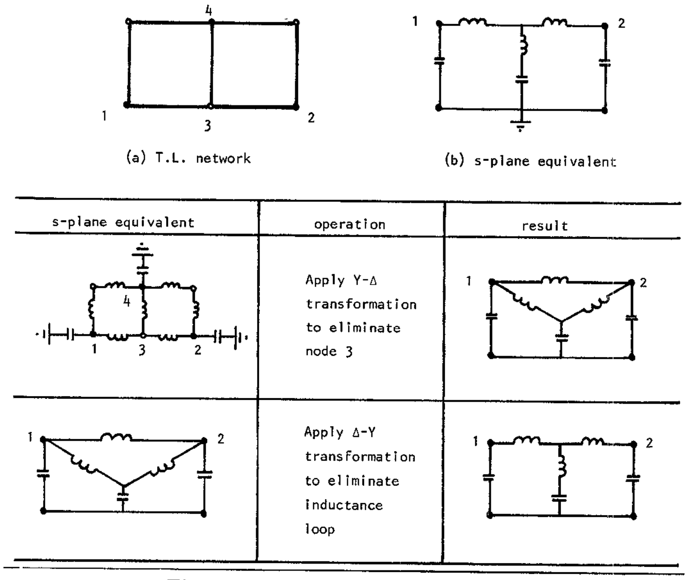


Fig. 8. Equivalent circuit derivation.

tion twice. Attempts by the author to simplify the network by existing graph-transformation technique failed. The failure seems to stem from the fact that there is no inductance conveniently nearby, which is a prior requirement for Sato's transformation (see [21]). Without the graph-transformation technique shown here, the derivation of two-port parameters of the given network by any algebraic mean is a formidable task.

In most examples above, the number of operations required to arrive at the final result is about half of those required before [3]. The number of basic identities employed is also half of the original number required (Table I).

V. Conclusion

Apart from simplifying the graph transformation, the s-plane equivalents shown in Section IV have the added advantage that they are in mid-series low-pass ladder form, with all their element values positive. The networks, therefore, can be synthesized directly by the classical zero-shifting procedure [17].

The only restriction for the s-plane presentation to be viable is that TL networks must be *normal*. It has been shown that this is also the condition for driving-point immittance to be a positive real rational function in λ (Theorem 3). This condition can always be met if the TL network is designed by insertion-loss method. (See footnotes 5 and 6.)

In summary, we have shown the following.

1) The s-plane presentation is possible for a sufficiently general class of networks of interest.

2) It simplifies graph-transformation techniques and reveals additional physical insights.

As such, the s-plane presentation thus constitutes an effective new tool for analysis and synthesis of TL networks.

Appendix I
Proof of Theorem 1

It is clear that Theorem 1 would follow if it could be shown that Y_1 and Y_2 are rational PR matrices in λ. $Y_1(Y_2)$ is the n-port DP admittance matrix looking into one side of $\mathfrak{N}_l$ with the other side terminated in $\mathfrak{N}_2$ ($\mathfrak{N}_1$), as shown in Fig. 2. Because of symmetry, we need only to show one admittance matrix (Y_1) to be rational PR.

Let the $n \times n$ matrix Z_0 denote the characteristic impedance matrix of n unit lines of $\mathfrak{N}_l$. This characteristic impedance matrix must be positive definite, but not necessarily diagonal, as the n unit lines may be coupled.

The $2n \times 2n$ scattering matrix of $\mathfrak{N}_l$, normalizing with respect to $2n \times 2n$ positive definite matrix $Z_0 \dotplus Z_0$, is

$$S_l(\lambda) = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} = \begin{pmatrix} 0_n & e^{-\tau p}1_n \\ e^{-\tau p}1_n & 0_n \end{pmatrix} \cdots \tag{8}$$

where all the submatrices in (8) are of dimension $n \times n$.[7]

Since $\mathfrak{N}_2$ is a lumped *RLC* network in λ-plane, its $n \times n$ scattering matrix S_2, normalizing with respect to S_0, must be a bounded real *rational* matrix of λ. It can also be shown that S_2 always exists.

Consider the n-port network consisting of $\mathfrak{N}_l$ and $\mathfrak{N}_2$ in cascade. Its scattering matrix S_{l2}, normalizing with respect to Z_0, is given by [11, eq. 3–20, p. 59]

$$\begin{aligned} S_{l2}(\lambda) &= S_{11} + S_{12}S_2(1_n - S_{22}S_2)^{-1}S_{21} \\ &= e^{2\tau p}S_2 \\ &= \frac{1-\lambda}{1+\lambda} S_2(\lambda). \end{aligned} \tag{9}$$

Equation (9), together with the fact that $S_2(\lambda)$ is a rational bounded real matrix, implies the following.

1) $S_{l2}(\lambda)$ is a rational matrix in λ.
2) $S_{l2}(\lambda)$ is holomorphic in $\mathrm{Re}\,\lambda \geq 0$.
3) $1_n - S_{l2}^*(j\omega)S_{l2}(j\omega) = 1_n - S_2^*(j\omega)S_2(j\omega) \geq 0, \quad \forall \omega.$

By the fundamental theorem [11, p. 98], it follows that $S_{l2}(\lambda)$ is also a *rational* bounded real matrix in λ. This, in turn, implies that

$$Y_1 = Z_0^{-1/2}(1_n - S_{l2})(1_n + S_{l2})^{-1}Z_0^{-1/2}$$

must be a PR *rational* function in λ.

The proof is thus completed.

Appendix II
Proof of Theorem 2

For simplicity, we shall first limit our discussion to TL networks consisting of unit lines only; the main emphasis of Theorem 2 is in the existence of the two pulses. Towards this end, we need only to show the following.

a) Between any pair of sending and receiving nodes, there is at least one route consisting of an even number of unit lines (even route) and one route consisting of an odd number of unit lines (odd route).

[7] The nondiagonal nature of terminating the impedance matrix does not limit our formulation in any way. Interested readers are referred to [12, p. 266] for justification.

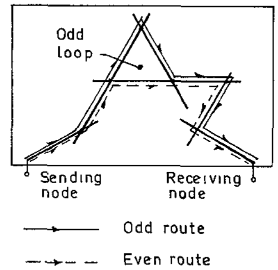


Fig. 9. Odd and even routes of nonnormal TL network.

b) Among those pulses taking even routes, there is at least one pulse that will survive possible cancellation on its arrival. Similarly, there is at least one surviving pulse taking the odd route.

Proof: Part a) is simple. If the TL network is not normal, there exists an odd loop in its TL graph. Since the graph is *connected*, there is at least one TL path joining the odd loop to the pair of sending and receiving nodes. If this route is even, then the other route forming the odd loop must be odd, and vice versa. This argument is shown graphically in Fig. 9.

To prove part b), we first note that the first pulse to arrive at the receiving node takes the shortest route. It cannot be reflected on its way[8] because it can only delay its arrival time by 2τ. Since the transmission coefficient of any idealized junction is always *positive*, the pulse which takes the route will always *preserve* its polarity. If the shortest route is nonunique, pulses taking different routes of the same route length would reenforce one another on arrival because of the same polarity. We can therefore argue that pulses taking the shortest even route and shortest odd route will both survive. The proof is thus completed.

If λ-plane *RLC* elements and multiwire lines are now included in the TL network, the theorem is still valid. In the proof above, we deal only with the first arrival pulses. Since pulses can pass through series or shunt *RLC* elements instantaneously without changing their polarity, the proof is still valid for the enlarged class of TL networks.

Acknowledgment

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Design of Microwave Filters by Sine-Plane Approach

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***Abstract*—Using a new sine-plane approach [7], an easy-to-use design procedure for microwave filters is developed. The design formulas are very simple (Tables I–III) and are valid for filters of wide bandwidths (Section V). Furthermore, the new design offers many advantages over other presently available designs.**

I. Introduction

MICROWAVE filters can be designed using two different approaches. They can be designed by approximation equations [1]–[3], and they can be designed by exact synthesis methods [4], [5]. Both approaches have their own merits. Explicit formulas are given in Cohn [1], Matthaei [2], and Cristal [3]; their design procedures are therefore easy to use. The design method presented by Wenzel [4] and Mumford [5] is exact, but this is achieved at the expense of greater numerical complexity.

With reference to the first approach, i.e., designing filters by approximation equations, Dishal [6] raised the following question: Why should one waste the space to put in rods #0 and #$(n+1)$, the only purpose of which is to properly couple the resistive generator and resistive load to the input–output resonances, respectively? This question has not been answered satisfactorily for parallel-coupled filters of wide bandwidths.

Using the new sine-plane approach presented in the companion paper [7], a new set of design equations will be derived in this paper. Apart from dissolving the objection raised above, the new design equations offer other advantages. These are discussed in the text of the paper.

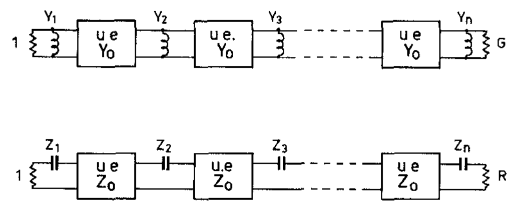

Fig. 1. Basic bandpass networks.

II. Theory

The theory derives mainly from the graphical transformation technique described in [7]. Through a series of transformations, it will be shown that microwave bandpass filter networks can be identified with lumped prototype filters of standard designs.

Consider the two basic bandpass filter structures in Fig. 1.[1] They are shown in Richards' λ-plane presentation,[2] and thus implicitly assumed that the networks consist of open-circuit stubs, short-circuit stubs, and unit transmission lines, all of which have the same electrical length. Note that all connect-

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[1] It can be shown that the interdigital filter and parallel-coupled filter are equivalent to one of the two network structures.

[2] $\lambda = \tanh \tau p$, where τ is the time taken for a pulse to traverse the unit length of transmission line, and $p = \sigma + j\omega$ is the complex frequency. Later, s and c will be used to denote $\sinh \tau p$ and $\cosh \tau p$, respectively.